IJCRR
Vol 04 issue 21
Section: General Science
Category: Research
Received on: 05/07/12
Revised on:02/08/12
Accepted on:18/08/12

ESTIMATION OF RELIABILITY IN MULTICOMPONENT STRESS-STRENGTH BASED ON GENERALIZED INVERTED EXPONENTIAL DISTRIBUTION

G. Srinivasa Rao

Department of Statistics, School of M and C S, Dilla University, Dilla, Ethiopia

E-mail of Corresponding Author: gaddesrao@yahoo.com

ABSTRACT

In this paper, we are mainly interested in estimating multicomponent stress- strength reliability. The system is regarded as alive only if at least s out of k (s<k) strengths exceed the stress. The reliability of such a system is obtained when strength, stress variates are given by generalized inverted exponential distribution with different shape parameters and common scale parameter. The research methodology adopted here is to estimate the parameters by using maximum likelihood estimation. The reliability is estimated using the maximum likelihood method of estimation when samples are drawn from strength and stress distributions. The reliability estimators are compared asymptotically. The results of small sample comparison of the reliability estimates are made through Monte-Carlo simulation. The simulation results indicates that the average bias and average MSE are decreases as sample size increases for both methods of estimation in reliability. The length of the confidence interval is also decreases as the sample size increases and coverage probability is close to the nominal value in all sets of parameters considered here. By using real data sets we well illustrate the procedure.

Key words: *Generalized inverted exponential distribution, reliability estimation, stress- strength, ML estimation, confidence intervals.*

INTRODUCTION

Recently the two-parameter generalized inverted exponential distribution (GIED) has been proposed and studied extensively by the Abouammoh and Alshingiti (2009). The GIED has the following density function

$$f(x;\alpha,\lambda)=\frac{\alpha\lambda}{x^2}e^{-\lambda/x}\left(1-e^{-\lambda/x}\right)^{\alpha-1}; \quad \text{for } x\ge 0 \tag{1}$$

and the distribution function

$$F(x;\alpha,\lambda)=1-\left(1-e^{-\lambda/x}\right)^{\alpha}; \quad \text{for } x\ge 0. \tag{2}$$

The reliability function (rf) of GIED is given by

$$R(x)=\left(1-e^{-\lambda/x}\right)^{\alpha}; \quad \text{for } x\ge 0. \tag{3}$$

Here $\alpha>0$ and $\lambda>0$ are the shape and the scale parameters respectively. Now onwards GIED with the shape parameter α and scale parameter λ will be denoted by GIED(α,λ). It is noted

that the GIED(α,λ) is reduced to inverted exponential distribution (IED) for α =1. Furthermore, the distribution of $T=(1/X)$ has a generalized exponential distribution (GED) introduced by Gupta and

Kundu (1999). Abouammoh and Alshingiti (2009) investigated the statistical and reliability properties of GIED. They have studied the reliability properties of Equation (3) using different methods of estimation of parameters. It is noted that, Nadarajah and Kotz (2003) have discussed some properties of GIED. This generalization is called, in their paper, the exponentiated Frechet distribution.

The purpose of this paper is to study the reliability in a multicomponent stress-strength based on X, Y being two independent random variables, where $X \sim GIED(\alpha,\lambda)$ and $Y \sim GIED(\beta,\lambda)$.

Let the random samples $Y, X_1, X_2, ...X_k$ be independent, G(y) be the continuous distribution function of Y and F(x) be the common continuous distribution function of $X_1, X_2, ...X_k$. The reliability in a multicomponent stress-strength model developed by Bhattacharyya and Johnson (1974) is given by

$$R_{s,k} = \mathrm{P}\left[\text{at least } s \text{ of the } (X_1, X_2, ...X_k) \text{ exceed } Y\right]$$

$$= \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} [1-F(y)]^i [F(y)]^{k-i} dG(y), \qquad (4)$$

where $X_1, X_2, ...X_k$ are identically indipendently distributed (iid) with common distribution function $F(x)$ and subjected to the common random stress *Y*. The probability in (4) is called reliability in a multicomponent stress-strength model [Bhattacharyya and Johnson (1974)]. The survival probabilities of a single component stress-strength version have been considered by several authors assuming various lifetime distributions for the stress-strength random variates. Enis and Geisser (1971), Downtown (1973), Awad and Gharraf (1986), McCool (1991), Nandi and Aich (1994), Surles and Padgett (1998), Raqab and Kundu (2005), Kundu and Gupta (2005& 2006), Raqab *et al* (2008), Kundu and Raqab (2009). The reliability in a multicomponent stress-strength was developed by Bhattacharyya and Johnson (1974), Pandey and Borhan Uddin (1985). Recently Rao and Kantam (2010) studied estimation of reliability in multicomponent stress-strength for the log-logistic distribution and Rao (2012) developed an estimation of reliability in multicomponent stress-strength based on generalized exponential distribution.

Suppose a system, with *k* identical components, functions if at least $s(1 \le s \le k)$ components simultaneously operate. In its operating environment, the system is subjected to a stress *Y* which is a random variable with distribution function $G(.)$. The strengths of the components, that is the minimum stresses to cause failure, are independently and identically distributed random variables with distribution function $F(.)$. Then, the system reliability, which is the probability that the system does not fail, is the function $R_{s,k}$ given in (4). The estimation of survival probability in a multicomponent stress-strength system when the stress and strength variates follow GIED is not paid much attention. Therefore, an attempt is made here to study the estimation of reliability in multicomponent stress-strength model with reference to GIED. The rest of the paper is organized as follows. In Section 2, we discussed the research methodology for expression of $R_{s,k}$ and develop a procedure for estimating it. More specifically, we obtain the maximum likelihood estimates of the parameters. The MLE are employed to obtain the asymptotic distribution and confidence intervals for $R_{s,k}$. The results of small sample comparisons made through Monte Carlo simulations are in Section 3. Also, using real data, we illustrate the estimation process. Finally, the conclusion and comments are provided in Section 4.

Research Methodology for Maximum Likelihood Estimator of $R_{s,k}$

Let $X \sim GIED(\alpha,\lambda)$ and $Y \sim GIED(\beta,\lambda)$ with unknown shape parameters α and β and common scale parameter λ, where X and Y are independently distributed. The reliability in multicomponent stress-strength for GIED using (4) we get

$$R_{s,k} = \sum_{i=s}^{k}\binom{k}{i}\int_0^\infty \left[1-\left(1-e^{-\lambda/y}\right)^{\alpha}\right]^i \left[\left(1-e^{-\lambda/y}\right)^{\alpha}\right]^{k-i} \frac{\lambda\beta}{y^2} e^{-\lambda/y}\left(1-e^{-\lambda/y}\right)^{\beta-1} dy$$

$$= \sum_{i=s}^{k}\binom{k}{i}\int_0^1 t^{\alpha(k-i)+\beta-1}\left(1-t^{\alpha}\right)^i \beta dt, \quad \text{where } t = 1-e^{-\lambda/y}$$

$$= \sum_{i=s}^{k}\binom{k}{i}\nu\int_0^1 z^{k-i+\nu-1}\left(1-z\right)^i dz, \quad \text{where } z = t^{\alpha},\ \nu = \frac{\beta}{\alpha}$$

$$= \sum_{i=s}^{k}\binom{k}{i}\nu\, \mathrm{B}(k-i+\nu, i+1).$$

After the simplification we get

$$R_{s,k} = \nu\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}(k+\nu-j)\right]^{-1} \quad \text{since } k \text{ and } i \text{ are integers.} \tag{5}$$

The probability in (5) is called reliability in a multicomponent stress-strength model. If α and β are not known, it is necessary to estimate α and β to estimate $R_{s,k}$. In this paper we estimate α and β by ML method. The estimates are substituted in ν to get an estimate of $R_{s,k}$ using equation (5). The theory of methods of estimation is explained below.

It is well known that the method of maximum likelihood estimation (MLE) has invariance property. We have proposed ML estimator for the reliability of multicomponent stress-strength model by considering the estimators of the parameters of stress and strength distributions by ML method of estimation in GIED. Let $X_1 < X_2 < .... < X_n$; $Y_1 < Y_2 < .... < Y_m$ be two ordered random samples of size n, m respectively on strength and stress variates each following GIED with shape parameters α and β respectively and common scale parameter λ. The log-likelihood function of the observed sample is

$$L(\alpha,\beta,\lambda) = (m+n)\ln\lambda + n\ln\alpha + m\ln\beta - \lambda\left[\sum_{i=1}^{n}\frac{1}{x_i} + \sum_{j=1}^{m}\frac{1}{y_j}\right] + (\alpha-1)\sum_{i=1}^{n}\ln(1-e^{-\lambda/x_i}) +$$

$$(\beta-1)\sum_{j=1}^{m}\ln(1-e^{-\lambda/y_j}) - \sum_{i=1}^{n}\ln x_i^2 - \sum_{j=1}^{m}\ln y_j^2 \tag{6}$$

The MLEs $\hat{\alpha}, \hat{\beta}$ and $\hat{\lambda}$ of α, β and λ are respectively can be obtained as the iterative solution of

$$\frac{\partial\, \mathrm{L}}{\partial\alpha} = 0 \;\Rightarrow \frac{n}{\alpha} + \sum_{i=1}^{n}\ln(1-e^{-\lambda/x_i}) = 0, \tag{7}$$

$$\frac{\partial\, \mathrm{L}}{\partial\beta} = 0 \;\Rightarrow \frac{m}{\beta} + \sum_{j=1}^{m}\ln(1-e^{-\lambda/y_j}) = 0, \tag{8}$$

$$\frac{\partial \mathrm{L}}{\partial \lambda}=0 \;\Rightarrow \frac{m+n}{\lambda}-\left[\sum_{i=1}^{n}\frac{1}{x_i}+\sum_{j=1}^{m}\frac{1}{y_j}\right]+(\alpha-1)\sum_{i=1}^{n}\frac{\frac{1}{x_i}e^{-\lambda/x_i}}{1-e^{-\lambda/x_i}}+(\beta-1)\sum_{j=1}^{m}\frac{\frac{1}{y_j}e^{-\lambda y_j}}{1-e^{-\lambda/y_j}}=0\,. \tag{9}$$

From (7), (8) and (9), we obtain

$$\hat{\alpha}=\frac{-n}{\sum_{i=1}^{n}\ln(1-e^{-\hat{\lambda}/x_i})}, \tag{10}$$

$$\hat{\beta}=\frac{-m}{\sum_{j=1}^{m}\ln(1-e^{-\hat{\lambda}/y_j})}, \tag{11}$$

and $\hat{\lambda}$ can be obtained as the solution of non-linear equation $g(\lambda)=0$

$$\Rightarrow \frac{m+n}{\lambda}-\frac{n\sum_{i=1}^{n}\frac{(1/x_i)e^{-\lambda/x_i}}{1-e^{-\lambda/x_i}}}{\sum_{k=1}^{n}\ln(1-e^{-\lambda/x_k})}-\frac{m\sum_{j=1}^{m}\frac{(1/y_j)e^{-\lambda/y_j}}{1-e^{-\lambda/y_j}}}{\sum_{k=1}^{m}\ln(1-e^{-\lambda/y_k})}-\sum_{i=1}^{n}\frac{(1/x_i)}{1-e^{-\lambda/x_i}}-\sum_{j=1}^{m}\frac{(1/y_j)}{1-e^{-\lambda/y_j}}=0\,. \tag{12}$$

Therefore, $\hat{\lambda}$ is simple iterative solution of non-linear equation $g(\lambda)=0$. Once we obtain $\hat{\lambda}$, $\hat{\alpha}$ and $\hat{\beta}$ can be obtained from (10) and (11) respectively. Therefore, the MLE of $R_{s,k}$ becomes

$$\hat{R}_{s,k}=\hat{\nu}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}(k+\hat{\nu}-j)\right]^{-1} \quad \text{where} \quad \hat{\nu}=\frac{\hat{\beta}}{\hat{\alpha}}. \tag{13}$$

To obtain the asymptotic confidence interval for $R_{s,k}$, we proceed as follows:

The asymptotic variance of the MLE is given by

$$\mathrm{V}(\hat{\alpha})=\left[E\left(-\frac{\partial^2 L}{\partial \alpha^2}\right)\right]^{-1}=\frac{\alpha^2}{n} \quad \text{and} \quad \mathrm{V}(\hat{\beta})=\left[E\left(-\frac{\partial^2 L}{\partial \beta^2}\right)\right]^{-1}=\frac{\beta^2}{m} \tag{14}$$

The asymptotic variance (AV) of an estimate of $R_{s,k}$ which is a function of two independent statistics (say) α,β is given by Rao (1973).

$$\mathrm{AV}(\hat{\mathrm{R}}_{s,k})=\mathrm{V}(\hat{\alpha})\left(\frac{\partial \mathrm{R}_{s,k}}{\partial \alpha}\right)^2+\mathrm{V}(\hat{\beta})\left(\frac{\partial \mathrm{R}_{s,k}}{\partial \beta}\right)^2. \tag{15}$$

Thus from Equation (15), the asymptotic variance of $\hat{\mathrm{R}}_{s,k}$ can be obtained.

To avoid the difficulty of derivation of $R_{s,k}$, we obtain the derivatives of $R_{s,k}$ for (*s*, *k*)=(1,3) and (2,4) separately, they are given by

$$\frac{\partial \hat{R}_{1,3}}{\partial \alpha}=\frac{3\hat{\nu}}{\hat{\alpha}(3+\hat{\nu})^2} \text{ and } \frac{\partial \hat{R}_{1,3}}{\partial \beta}=\frac{-3}{\hat{\alpha}(3+\hat{\nu})^2}\,.$$

$$\frac{\partial \hat{R}_{2,4}}{\partial \alpha}=\frac{12(7+2\hat{\nu})}{\hat{\alpha}\left[(3+\hat{\nu})(4+\hat{\nu})\right]^2} \text{ and } \frac{\partial \hat{R}_{2,4}}{\partial \beta}=\frac{-12(7+2\hat{\nu})}{\hat{\alpha}\left[(3+\hat{\nu})(4+\hat{\nu})\right]^2}\,.$$

Thus $\mathrm{AV}(\hat{\mathrm{R}}_{1,3})=\dfrac{9\hat{\nu}^2}{\left[(3+\hat{\nu})\right]^4}\left(\dfrac{1}{n}+\dfrac{1}{m}\right)$.

$$\mathrm{AV}(\hat{\mathrm{R}}_{2,4})=\frac{144\hat{\nu}^2(2\hat{\nu}+7)^2}{\left[(3+\hat{\nu})(4+\hat{\nu})\right]^4}\left(\frac{1}{n}+\frac{1}{m}\right).$$

As $n\to\infty, m\to\infty$, $\dfrac{\hat{R}_{s,k}-R_{s,k}}{\mathrm{AV}(\hat{\mathrm{R}}_{s,k})}\xrightarrow{d}\mathrm{N}(0,1)$,

and the asymptotic 95% confidence interval for $R_{s,k}$ is given by $\hat{R}_{s,k}\mp 1.96\sqrt{\mathrm{AV}(\hat{\mathrm{R}}_{s,k})}$.

The asymptotic 95% confidence interval for $R_{1,3}$ is given by

$$\hat{R}_{1,3}\mp 1.96\frac{3\hat{\nu}}{\left[(3+\hat{\nu})\right]^2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)},\ \text{where } \hat{\nu}=\hat{\beta}/\hat{\alpha}.$$

The asymptotic 95% confidence interval for $R_{2,4}$ is given by

$$\hat{R}_{2,4}\mp 1.96\frac{12\hat{\nu}(2\hat{\nu}+7)}{\left[(3+\hat{\nu})(4+\hat{\nu})\right]^2}\sqrt{\left(\frac{1}{n}+\frac{1}{m}\right)},\ \text{where } \hat{\nu}=\hat{\beta}/\hat{\alpha}.$$

RESULTS AND DATA ANALYSIS

Results from simulation study

In this sub section we present some results based on Monte-Carlo simulations to compare the performance of the $R_{s,k}$ using for different sample sizes. 3000 random sample of size 10(5)35 each from stress and strength populations are generated for (α,β) =(3.0,1.0), (2.5,1.0), (2.0,1.0), (1.5,1.0), (1.0,1.0), (1.0,1.5),(1.0,2.0), (1.0,2.5) and (1.0,3.0) as proposed by of Bhattacharyya and Johnson (1974). The MLE of scale parameter λ is estimated by iterative method and using λ the shape parameters α and β are estimated from (10) and (11). These ML estimators of α and β are then substituted in ν to get the multicomponent reliability for (*s*, *k*) = (1, 3), (2, 4). The average bias and average mean square error (MSE) of the reliability estimates over the 3000 replications are given in Tables 1 and 2. Average confidence length and coverage probability of the simulated 95% confidence intervals of $R_{s,k}$ are given in Tables 3 and 4. The true value of reliability in multicomponent stress-strength with the given combinations of (α,β) for (*s*, *k*) = (1, 3) are 0.900, 0.882, 0.857, 0.818, 0.750, 0.667, 0.600, 0.545, 0.500 and for (*s*, *k*) = (2, 4) are 0.831, 0.802, 0.762, 0.701, 0.600, 0.485, 0.400, 0.336, 0.286. Thus the true value of reliability in multicomponent stress-strength decreases as β increases for a fixed α whereas reliability in multicomponent stress-strength increases as α increases for a fixed β in both the cases of (*s*, *k*). Therefore, the true value of reliability is decreases as ν increases and vice versa. The average bias and average MSE are decreases as sample size increases for both cases of estimation in reliability. Also the bias is negative in most of the combinations of the parameters in both situations of (*s*, *k*). It verifies the consistency property of the MLE of $R_{s,k}$. Whereas, among the parameters the absolute bias and MSE are increases as β increases for a fixed α in both the cases of (*s*, *k*) and the absolute bias and MSE are decreases as α increases for a fixed β in both the cases of (*s, k*). The length of the confidence interval is also decreases as the sample size increases. The coverage probability is close to the nominal value in all cases but less than 0.95. Overall, the performance of the confidence interval is quite good for all combinations of parameters. Whereas, among the parameters we observed the same phenomenon for average length and average coverage probability that we observed in case of average bias and MSE.

Data Analysis

In this sub section we analyze two real data sets and demonstrate how the proposed methods can be used in practice. The first data set reported by Lawless (1982) and second data set given by Fuller *et al* (1994) and these data sets are analyzed and fitted for various lifetime distributions. We fit the generalized inverted exponential distribution to the two data sets separately. The first data set (Lawless (1982); page 228) presented here arose in tests on endurance of deep groove ball bearings. The data presented are the number of million revolutions before failure for each of the 23 ball bearings in the life test and they are (Y): 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04 and 173.40.

The second data set was given by Fuller *et al* (1994) represents the data to predict the lifetime for a glass airplane window. The data are as follows (X): 18.83, 20.8, 21.657, 23.03, 23.23, 24.05, 24.321, 25.5, 25.52, 25.8, 26.69, 26.77, 26.78, 27.05, 27.67, 29.9, 31.11, 33.2, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, and 45.381. Abouammoh and Alshingiti (2009) studied the validity of the model for both data sets and they showed that by using different fitting procedures GIED fits quite well for both the data sets.

We use the iterative procedure to obtain the MLE of λ using (12) and MLEs of α and β are obtained by substituting MLE of λ in (10) and (11). The final estimates for real data sets are $\hat{\alpha}$ = 75.047698, $\hat{\beta}$ = 6.145771 and $\hat{\lambda}$ =141.565031. Base on estimates of α and β the MLE of $R_{s,k}$ become $\hat{R}_{1,3}$ =0.973428 and $\hat{R}_{2,4}$ = 0.953899. The 95% confidence intervals for $R_{1,3}$ become (0.959478, 0.987378) and for $R_{2,4}$ become (0.929909, 0.977889).

CONCLUSIONS

In this paper, we have studied the multicomponent stress-strength reliability for generalized inverted exponential distribution when both of stress, strength variates follows the same population. Also, we have estimated asymptotic confidence interval for multicomponent stress-strength reliability. The simulation results indicates that the average bias and average MSE are decreases as sample size increases for both methods of estimation in reliability. Among the parameters the absolute bias and MSE are increases (decreases) as β increases (α increases) in both the cases of (*s, k*). The length of the confidence interval is also decreases as the sample size increases and coverage probability is close to the nominal value in all sets of parameters considered here. Using real data, we illustrate the estimation process.

ACKNOWLEDGEMENT

Authors acknowledge the immense help received from the scholars whose articles are cited and included in references of this manuscript. The authors are also grateful to authors / editors /publishers of all those articles, journals and books from where the literature for this article has been reviewed and discussed.

REFERENCES

1. Abouammoh, A.M. and Alshingiti, A.M. (2009). Reliability estimation of generalized inverted exponential distribution, Journal of Statistical Computation and Simulation, 79(11), 1301-1315.

2. Awad, M. and Gharraf, K. (1986). Estimation of $P(Y < X)$ in Burr case: A comparative study, Commun. Statist. - Simula. & Comp., 15, 389-403.
3. Bhattacharyya, G.K. and Johnson, R.A. (1974). Estimation of reliability in multicomponent stress-strength model, JASA, 69, 966-970.
4. Downtown, F. (1973). The estimation of $P(X > Y)$ in the normal case, Technometrics, 15, 551-558.
5. Enis, P. and Geisser, S. (1971). Estimation of the probability that $Y < X$, JASA, 66, 162-168.
6. Fuller Jr, E.R., Frieman, S.W., Quinn, J.B., Quinn, G.D. and Carter, W.C. (1994). Fracture mechanics approach to the design of glass aircraft window: a case study, SPIE Proc. 2286, 419-430.
7. Gupta, R.D. and Kundu, D. (1999). Generalized exponential distributions. Australian and New Zealand Journal of Statistics, 41, 173-188.
8. Kundu, D. and Gupta, R.D. (2005). Estimation of $P(Y < X)$ for the generalized exponential distribution, Metrika, 61 (3), 291-308.
9. Kundu, D. and Gupta, R.D. (2006). Estimation of $P(Y < X)$ for Weibull distribution, IEEE Transactions on Reliability, 55 (2), 270-280.
10. Kundu, D. and Raqab, M.Z. (2009). Estimation of R= $P(Y < X)$ for three-parameter Weibull distribution, Statistics and Probability Letters, 79, 1839-1846.
11. Lawless J.F. (1982). Statistical Models and Methods for Lifetime Data. New York: John Wiley & Sons.
12. McCool, J. I. (1991). Inference on $P(Y < X)$ in the Weibull case, Commun. Statist. - Simula. & Comp., 20, 129-148.
13. Nadarajah, S. and Kotz, S. (2003). The exponentiated Frechet distribution, Available at: Interstat.statjournals.net, 0312001.
14. Nandi, S.B. and Aich, A.B. (1994). A note on estimation of $P(X > Y)$ for some distributions useful in life- testing, IAPQR Transactions, 19(1), 35-44.
15. Pandey, M. and Borhan Uddin, Md. (1985). Estimation of reliability in multicomponent stress-strength model following Burr distribution, Proceedings of the First Asian congress on Quality and Reliability, New Delhi, India, 307-312.
16. Rao, C. R. (1973). Linear Statistical Inference and its Applications, Wiley Eastern Limited, India.
17. Rao, G.S. (2012). Estimation of reliability in multicomponent stress-strength model based on generalized exponential distribution, Colombian Journal of Statistics, 35(1), 67-76.
18. Rao, G.S. and Kantam, R.R.L. (2010). Estimation of reliability in multicomponent stress-strength model: log-logistic distribution, Electronic Journal of Applied Statistical Analysis, 3(2), 75-84.
19. Raqab, M.Z. and Kundu, D. (2005). Comparison of different estimators of $p(Y < X)$ for a scaled Burr type X distribution, Commun. Statist. - Simula. & Comp., 34 (2), 465-483.
20. Raqab, M.Z., Madi, M.T. and Kundu, D. (2008). Estimation of $P(Y < X)$ for the 3-parameter generalized exponential distribution, Commun. Statist - Theor. Meth., 37 (18), 2854-2864.
21. Surles, J.G. and Padgett, W.J. (1998). Inference for $P(Y < X)$ in the Burr type X model, Journal of Applied Statistical Sciences, 7, 225-238.

Table 1. Average bias of the simulated estimates of $R_{s,k}$

(s,k)	(n,m)	(α,β)								
		(3.0,1.0)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)	(1.0,3.0)
(1,3)	(10,10)	-0.00093	-0.00144	-0.00288	-0.00541	-0.00956	-0.01393	-0.01688	-0.01901	-0.02009
	(15,15)	-0.00083	-0.00132	-0.00243	-0.00410	-0.00694	-0.01011	-0.01291	-0.01365	-0.01446
	(20,20)	-0.00040	-0.00084	-0.00178	-0.00313	-0.00564	-0.00839	-0.01038	-0.01143	-0.01223
	(25,25)	0.00038	-0.00077	-0.00135	-0.00126	-0.00484	-0.00783	-0.00701	-0.00809	-0.00973
	(30,30)	-0.00024	-0.00067	-0.00111	-0.00202	-0.00368	-0.00558	-0.00697	-0.00793	-0.00853
	(35,35)	-0.00015	-0.00065	-0.00101	-0.00203	-0.00336	-0.00493	-0.00601	-0.00665	-0.00705
(2,4)	(10,10)	0.00075	-0.00158	-0.00243	-0.00453	-0.00761	-0.00919	-0.00922	-0.00866	-0.00743
	(15,15)	0.00069	-0.00152	-0.00238	-0.00411	-0.00641	-0.00792	-0.00880	-0.00767	-0.00687
	(20,20)	0.00051	-0.00095	-0.00166	-0.00315	-0.00541	-0.00687	-0.00741	-0.00684	-0.00633
	(25,25)	0.00026	0.00090	0.00031	-0.00264	-0.00491	-0.00508	-0.00515	-0.00524	-0.00495
	(30,30)	0.00020	-0.00047	-0.00114	-0.00207	-0.00361	-0.00480	-0.00508	-0.00515	-0.00485
	(35,35)	0.00012	-0.00040	-0.00107	-0.00222	-0.00337	-0.00425	-0.00451	-0.00425	-0.00388

Table 2. Average MSE of the simulated estimates of $R_{s,k}$

(s,k)	(n,m)	(α,β)								
		(3.0,1.0)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)	(1.0,3.0)
(1,3)	(10,10)	0.00214	0.00278	0.00379	0.00567	0.00875	0.01222	0.01431	0.01556	0.01610
	(15,15)	0.00137	0.00176	0.00230	0.00331	0.00517	0.00733	0.00887	0.00953	0.00991
	(20,20)	0.00099	0.00128	0.00182	0.00252	0.00392	0.00569	0.00672	0.00747	0.00777
	(25,25)	0.00078	0.00101	0.00137	0.00198	0.00316	0.00448	0.00537	0.00589	0.00618
	(30,30)	0.00063	0.00082	0.00112	0.00157	0.00245	0.00349	0.00441	0.00473	0.00496
	(35,35)	0.00057	0.00073	0.00096	0.00138	0.00213	0.00307	0.00364	0.00406	0.00426
(2,4)	(10,10)	0.00552	0.00694	0.00900	0.01229	0.01639	0.01883	0.01863	0.01766	0.01594
	(15,15)	0.00356	0.00446	0.00556	0.00745	0.01016	0.01197	0.01223	0.01132	0.01027
	(20,20)	0.00258	0.00324	0.00441	0.00571	0.00777	0.00939	0.00936	0.00884	0.00800
	(25,25)	0.00205	0.00258	0.00335	0.00452	0.00632	0.00751	0.00766	0.00721	0.00659
	(30,30)	0.00167	0.00209	0.00276	0.00361	0.00495	0.00588	0.00630	0.00583	0.00532
	(35,35)	0.00150	0.00187	0.00237	0.00317	0.00431	0.00519	0.00527	0.00503	0.00460

Table 3. Average confidence length of the simulated 95% confidence intervals of $R_{s,k}$

(s,k)	(n,m)	(α,β)								
		(3.0,1.0)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)	(1.0,3.0)
(1,3)	(10,10)	0.15477	0.17904	0.21163	0.25690	0.32174	0.37629	0.40157	0.41040	0.41005
	(15,15)	0.12794	0.14784	0.17447	0.21193	0.26595	0.31242	0.33453	0.34303	0.34366
	(20,20)	0.11064	0.12789	0.15111	0.18376	0.23107	0.27188	0.29176	0.29937	0.30027
	(25,25)	0.09850	0.11384	0.13453	0.16364	0.20596	0.24320	0.26147	0.26908	0.27033
	(30,30)	0.09064	0.10471	0.12360	0.15028	0.18916	0.22327	0.23986	0.24689	0.24802
	(35,35)	0.08407	0.09706	0.11472	0.13932	0.17528	0.20690	0.22261	0.22909	0.23027
(2,4)	(10,10)	0.24901	0.28361	0.32770	0.38338	0.44890	0.47978	0.47263	0.44882	0.42005
	(15,15)	0.20686	0.23553	0.27214	0.31923	0.37511	0.40282	0.39744	0.37903	0.35501
	(20,20)	0.17945	0.20444	0.23637	0.27783	0.32740	0.35228	0.34869	0.33249	0.31161
	(25,25)	0.16006	0.18239	0.21111	0.24829	0.29318	0.31704	0.31471	0.30093	0.28254
	(30,30)	0.14736	0.16786	0.19408	0.22823	0.26956	0.29137	0.28874	0.27604	0.25898
	(35,35)	0.13672	0.15567	0.18025	0.21174	0.25009	0.27043	0.26856	0.25676	0.24110

Table 4. Average coverage probability of the simulated 95% confidence intervals of $R_{s,k}$

(s,k)	(n,m)	(α,β)								
		(3.0,1.0)	(2.5,1.0)	(2.0,1.0)	(1.5,1.0)	(1.0,1.0)	(1.0,1.5)	(1.0,2.0)	(1.0,2.5)	(1.0,3.0)
(1,3)	(10,10)	0.92600	0.92767	0.93100	0.93067	0.92700	0.92100	0.91133	0.90700	0.89833
	(15,15)	0.92467	0.92933	0.93300	0.93500	0.94167	0.93500	0.92767	0.92100	0.91500
	(20,20)	0.93100	0.93333	0.93267	0.93733	0.93800	0.93567	0.92867	0.91933	0.91500
	(25,25)	0.93233	0.93533	0.93767	0.93700	0.93867	0.93800	0.93333	0.92667	0.91933
	(30,30)	0.93000	0.93600	0.93567	0.93967	0.94133	0.93733	0.93033	0.93033	0.92500
	(35,35)	0.93367	0.93633	0.93733	0.94467	0.94633	0.94267	0.93800	0.92900	0.92367
(2,4)	(10,10)	0.92467	0.92533	0.92767	0.92567	0.92533	0.91800	0.91267	0.91367	0.91367
	(15,15)	0.92100	0.92833	0.93100	0.93333	0.93633	0.93100	0.93000	0.92900	0.92600
	(20,20)	0.93033	0.93300	0.93033	0.93600	0.93700	0.93733	0.93267	0.92133	0.91833
	(25,25)	0.93133	0.93400	0.93400	0.93833	0.93767	0.93700	0.93367	0.92767	0.92167
	(30,30)	0.92933	0.93433	0.93433	0.93933	0.93967	0.93867	0.93267	0.93267	0.92767
	(35,35)	0.93300	0.93667	0.93800	0.94233	0.94600	0.94133	0.93600	0.93200	0.92667